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A Numerical and Analytic Analysis of Nonlinear Implicit Differenital Equations Arising in Control and Circuit Problems

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The major objective of AFOSR-87-0051 was to develop analytic and numerical methods for implicit systems of differential equations arising in control and circuit problems. This report describes progress made during the period of funded research.

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January 15, 1987 to January 14, 1991

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January 9, 1991

1   Abstracts

1.1   Research Problem

Systems of ordinary differential equations are widely used throughout science and engineering to describe varying physical phenomena. Often these differential equations are initially *implicit*, that is, the highest derivative is not solved for. Traditionally, it has been necessary to first make the equations explicit before the system could be simulated or analyzed. This research is part of the current research effort under way in several disciplines to be able to carry out many parts of design and analysis with these original implicit equations. The ability to do this would not only reduce design and analysis time, but also make possible the consideration of more sophisticated models which in turn will provide enhanced system performance.

1.2   Accomplishments

During the funded period several significant advances have been achieved.

The first general method for the solution of solvable higher index nonlinear implicit systems of differential equations has been introduced and some of its properties developed. Applications to both optimal and prescribed trajectory control have been considered. Specific applications have included chemical reactors, robotic arms, and

shuttle re-entry. In addition, algorithms for the determination of control properties such as observability and controllability directly from the original implicit model have been developed for linear time varying systems. These algorithms are superior to previous algorithms even in the explicit case. The extension of these algorithms to nonlinear control systems has begun.

2 Research Objectives

This research project was to develop methods for the numerical and analytic analysis of implicit systems of differential equations,

$$\text{(DAE)} \quad F(x', x, t) = 0 \tag{1}$$

which are not equivalent to an explicit ordinary differential equation (ODE),

$$\text{(ODE)} \quad x' = G(x, t) \tag{2}$$

That is, the Jacobian $F_{x'}$ of (1) is identically singular. Depending on the type of application, the system (1) is also called a descriptor, semi-state, singular, noncanonic, generalized state space, or differential-algebraic equation (DAE) system. Such systems arise in many circuit and control problems and when solving partial differential equations by the method of lines [12]. Many mechanics and engineering problems are initially formulated in the implicit form (1). Frequently, considerable effort must be expended to derive an explicit model from the original implicit model. These difficulties can be increased by such factors as closed kinematic loops in mechanics (vehicular sytems and constrained robotic arms), or multiple reference frames in flight trajectory control. In design problems, different values of design or model parameters may lead to different explicit models. Also, going from an implicit to an explicit system may destroy the sparsity and structure of the equations and produce models in terms of nonphysical variables. Consequently there has been an increasing interest in recent years in such diverse areas as chemical engineering, multibody dynamics, flight control, and electrical engineering (to name a few) in working directly with the original implicit differential equations [12].

The *index*, k, of a singular system is one measure of how singular the system is. Implicit and explicit ODE's are index zero. Index one DAE systems are usually easily handled. However, fully implicit nonlinear index one systems are still of some research interest. Higher index ($k \geq 2$) systems frequently arise in problems ranging from prescribed path (trajectory) control to constrained mechanical systems such as those modeling robotic and vehicular systems. Throughout the research period we

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have used these applications to motivate and shape our results. These higher index problems are both numerically and analytically much more difficult to work with and were the systems of interest in this project. The specific proposed research topics were as follows.

Objective 1. Develop and analyze numerical algorithms for the solution of higher index implicit systems of ordinary differential equations. Procedures are to be designed to work with (1) in its original implicit form and not require coordinate changes. Particular emphasis will be placed on solving those implicit systems for which standard numerical methods for DAEs such as backward differentiation formulas (BDF) or implicit Runge-Kutta (IRK) do not converge. These classical methods will also be considered as appropriate.

Objective 2. Characterize consistent initial conditions and determine ways to compute them. These initial values can then be used with the new methods of Objective 1, or with classical methods.

Objective 3. Develop explicit existence results and solution formula for additional classes of higher index singular systems. These can be used to both test numerical algorithms and suggest new analytical results.

Objective 4. Develop more general existence and uniqueness results based directly on general assumptions on (1). Most existing results require first performing nonlinear time varying coordinate changes.

Objective 5. Examine the linearization and approximation of nonlinear singular control problems so that the linear theory already developed for linear time invariant control systems [O2] can be applied to real life nonlinear control problems.

Objective 6. Examine the regularization problem for higher index systems and its relationship to problems in the numerical approximation of higher index singular systems.

Objective 7. Use the preceding results to examine the relationships between the properties of the discrete control systems that arise when discretizing singular time varying continuous control systems, say by BDF, and the properties of the original continuous system.

Objective 8. Apply the preceding results to the study of ill-conditioning in nearly higher index singular systems.

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Objective 9. Examine how the formulation and structure of control and circuit problems can be exploited to give more efficient algorithms and stronger analytical results.

Objective 10. Study noisy descriptor systems and their relationship to the numerical properties of algorithms for solving singular systems.

3 Results of Funded Research

As part of the research funded by AFOSR-84-0240, the principal investigator developed analytical structure theorems and a general numerical procedure for the linear time varying DAE

$$E(t)x'(t) + F(t)x(t) = f(t) \tag{3}$$

This numerical algorithm was the first general numerical procedure for (3). The analytical results were of wider applicability then those that had previously existed.

During the current funding period, progress has been made on several fronts.

3.1 General Numerical Method

In [4] the first general numerical method has been proposed for (1). This method is a generalization and modification of that developed for (3). Enough analysis was carried out in [4] to show that the method works in principle. The results of initial numerical testing on a chemical reactor control problem are in agreement with these theoretical results [Objectives 1,2]. Additional development of the numerical method is underway. The prescribed path control of a two link robot arm with flexible joints is examined in [4], [11]. It is shown there how our approach can be used to compute consistent initial conditions and feedback laws giving the control. The control problem structure is exploited to reduce the computational effort. Ways to reduce the number of Newton iterations at each time step are examined. This problem is of special interest because it is index five and cannot be analyzed by classical methods without time variable coordinate changes intermixed with differentiations [Objectives 1,2,9].

During the funding period, the principal investigator received grants totalling 30 units of computer time from the Cornell National Supercomputer Facility (CNSF) to pursue this research. Most of the calculations for the robotic path control problem were performed on the IBM 3090 at the CNSF.

Differentiation of constraints has been used with DAEs for over 20 years. This differentiation may be used to reduce the index or as part of a control theoretic procedure. However, the potential consequences of doing this differentiation had not been carefully studied. In [16] we examined the numerical and analytic consequences of

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constraint differentiation and established when numerical difficulties would be introduced [Objectives 1,5,6,8].

The approach of [4] can be viewed as a *completion* of a vector field only partially explicitly defined by the implicit system (1). Equivalently this method imbeds the solutions of the DAE (1) into the solutions of an ODE (2). The research begun in [16] was continued in [17], [18]. These papers begin to lay out the theoretical footing needed to completely analyze the general method of [4] and develop improved implementations.

3.2 Classical Numerical Methods

The general procedures described in Section 2.1 have the advantage that they work on all solvable DAE's. However, they tend to be computationally intensive. For some important classes of applications, particularly some of those in robotics and mechanics, it has been possible to modify classical numerical methods, such as BDF and IRK methods, to compute solutions. This work is still underway. The paper [1] develops a structure which can sometimes be exploited when considering BLᵀ methods and showed additional classes of systems for which BDF methods converge. This, and other papers, introduced several types of structures for (3). In [2] we showed how all these structures could be combined into one large type of structure which implied solvability and convergence of BDF methods. Nonlinear versions of these generalized cascades were also discussed in [2], [Objectives 1,3,4,9].

In [6] we showed that the boundary layer of numerical nonconvergence observed for BDF methods on higher.index systems was directly related to the distributional solutions for inconsistent initial conditions. We also showed convergence of the numerical solutions, in a weak sense, within the boundary layer. This provided a nice unification of several previously unrelated ideas [Objectives 5,6,7,8].

With the increasing availability of DAE codes, it is expected that some users may attempt to use these codes on problems for which the codes were not originally designed. The possible consequences of soing this had not previously been examined. In [11] we give an example of a nonsolvable linear time varying DAE for which a first order BDF method converges to a different solution then a second order BDF method does for all possible starting values for either method. Thus these numerical methods can converge to a unique limit even when solutions of the DAE are not unique [Objective 1].

We have also helped develop a much better understanding of the classical numerical methods in [12]. This work will not only make the DAE literature available in a coherent fashion to practitioners for the first time but also contains a substantial number of new analytical and numerical results relating the different methods. The

invited survey paper [13] also helps with the transfer of this knowledge to control engineers [Objectives 1,2,3].

3.3 Control Problems

The numerical methods discussed in the previous two sections have direct application to the simulation of control problems as reported in [11], [12]. However, we have also considered various control questions for physical systems modeled by DAEs. There are many specific control problems such as stabilization, disturbance decoupling, construction of observers, and realizations, that need to be carried out in specific applications. In [3], we showed how the numerical approach developed earlier for (3) can be used to give numerical algorithms for constructing local realizations for (3). This approach has the advantage that the only differentiated quantities are the given coefficients $E(t), F(t)$ and input $f(t)$ and all numerical calculations are only algebraic in nature [Objectives 3,9].

In the note [5] we cleared up some confusion and incorrect assertions in the literature on the controllability of descriptor systems. The paper [7] provided the first analysis of the method of orthogonal functions that had been proposed by other researchers in the engineering literature. It was shown for the first time that this method can fail to work for systems of index three or higher [Objective 3].

In [6] we began the investigation of the relationship between the numerical method of [3], [4] and certain concepts in geometric control theory. These results already point to a better understanding of the method of [4], an extension of some of the ideas of nonlinear control theory [O3] to more general classes of systems, and better numerical methods for computing some of the mathematical objects of interest in nonlinear control theory [Objectives 1,4,9]. The development of these ideas should make the results from nonlinear geometric control theory more accessible and more easily implemented by practitioners. In [9] we answered some questions on systems inversion.

Fundamental to many control procedures is the concept of observability. In [15] we introduced a general theory for observability of time varying DAEs in input-output form,

$$E(t)x'(t) + F(t)x(t) = B(t)u(t) \tag{4a}$$
$$y(t) = C(t)x(t) \tag{4b}$$

where $E(t)$ was singular and $C(t), E(t), F(t), B(t)$ could all have variable rank. All results were developed so that the only symbolic operations required were differentiations of the original coefficients. All subsequent calculations could be done numeri-

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cally. External descriptions were also developed. These results provided better computational algorithms for linear time varying systems even in the classical case when E was nonsingular [Objectives 4,9]. In [19], [20] this theory, and the computational algorithms, were extended to include controllability. The duality of controllability and observability for sytems of the form (4) was also developed in [20].

4 Impact of Research Results

Research on implicit models is already having an effect on engineering practice. The earlier work on index one systems has lead to the development of such productions numerical codes as DASSL [12] which are being widely used. Our research on higher index problems, combined with that of other scientists around the world, has reached the point where dependable higher index numerical codes can be expected within the next few years. These codes will not only have a significant impact on control engineering but on many other areas of technology.

5 Publications

The following papers have been written as part of this research project.

[1] K. D. Clark, *The numerical solution of some higher index time-varying semistate systems by difference methods*, Circuits Systems & Signal Processing, 6 (1987), 61-75.

[2] S. L. Campbell and K. D. Clark, *Singular control problem structure and the convergence of backward differentiation formulas*, in Linear Circuits, Systems and Signal Processing, edited by C. I. Brynes, C. F. Martin, and R. E. Saeks, North Holland, (1988), 19-26.

[3] S. L. Campbell, *Local realizations of time varying descriptor systems*, Proc. 26th IEEE Conference on Decision and Control, (1987), 1129-1130.

[4] S. L. Campbell, *A computational method for general higher index singular systems of differential equations*, IMACS Transactions on Scientific Computing, (1990), Vol 1.2, 555-560.

[5] S. L. Campbell, *A comment on controlling generalized state-space (descriptor) systems*, Int. J. Control, 46 (1987), 2229-2230.

[6] S. L. Campbell, *Distributional convergence of BDF approximations to solutions of descriptor systems*, Circuits Systems & Signal Processing, 8 (1989), 261-265.

[7] S. L. Campbell and K. D. Yeomans, *Solving singular systems using orthogonal functions*, IEE Proc., 137 (1990), 222-224.

[8] S. L. Campbell and W. J. Terrell, *Derivative arrays, geometric control theory, and realizations of linear descriptor systems*, 1987 International Symposium on Singular Systems, Atlanta, GA, (1987), 10-13.

[9] S. L. Campbell, *Comments on functional reproducibility of time-varying input-output systems*, IEEE Transactions on Automatic Control, 34 (1989), 922-924.

[10] S. L. Campbell and K. D. Clark, *Convergence of BDF approximations for non-solvable differential algebraic equations*, Applied Numerical Mathematics, 6 (1989/1990), 153-158.

[11] S. L. Campbell, *A general method for nonlinear descriptor systems: an example from robotic path control*, Proc. 27*th* IEEE Conference on Decision and Control, (1988), 630-631.

[12] K. E. Brenan, S. L. Campbell, and L. R. Petzold, *Numerical Solution of Initial Value Problems in Differential Algebraic Equations*, Elsevier, 1989, 210 pages.

[13] S. L. Campbell, *The numerical solution of singular systems arising in control problems*, Proc. American Control Conference, (1989), 2128-2133.

[14] S. L. Campbell, *Comments on 2-D descriptor systems*, Automatica (to appear).

[15] S. L. Campbell and W. J. Terrell, *Observability of linear time varying descriptor systems*, SIAM J. Matrix Analysis, (to appear).

[16] S. L. Campbell and B. Leimkuhler, *Differentiation of constraints in differential algberaic equations*, preprint, 1989.

[17] S. L. Campbell, *Uniqueness of completions for linear time varying differential algebraic equations*, Linear Algebra & Its Applications (to appear).

[18] S. L. Campbell, *Least squares completions of nonlinear higher index DAE control problems*, preprint, 1990.

[19] S. L. Campbell, N. K. Nichols, and W. J. Terrell, *Duality, observability, and controllability for linear time varying descriptor systems*, Circuits Systems & Signal Processing (to appear).

[20] W. J. Terrell, *Observability and External Description of Linear Time Varying Singular Control Systems*, Ph.D. Thesis, North Carolina State University, Raleigh, 1990.

[21] S. L. Campbell, *Descriptor Systems in the 90's*, Proc. 29*th* IEEE Conference on Decision & Control, (1990), 442-447.

6 Professional Personnel

The following personnel have been associated with this project.

1. Dr. Stephen L. Campbell, Professor of Mathematics at North Carolina State University, is the principal investigator.

2. Lt. Kevin D. Yeomans wrote his masters project under the direction of the principal investigator. His project was titled, *Use of Orthogonal Functions with Singular Systems*, and resulted in the publication [7]. He is currently on active duty with the United States Air Force.

3. Dr. William J. Terrell was a graduate student supported by funds from AFOSR 87-0051. He wrote his thesis, *Observability and External Description of Linear Time Varying Singular Control Systems*, under the direction of the principal investigator and received his Ph.D. in Applied Mathematics on Dec. 19, 1990, from North Carolina State University.

4. Ms. Wang Li was a masters student whose project, *The Numerical Solution of High Index Nonlinear Singular Systems of Differential Equations*, was directed by the principal investigator. The project involved a numerical implementation of some of the algorithms in [4].

5. Mr. Edward Moore is a current Ph.D. student of the principal investigator. He is investigating the numerical implimentation of some of the general methods developed as part of this project.

7 Interactions

7.1 Oral Presentations

The following oral presentations have been made as part of this research project.

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1. The principal investigator presented the invited paper [2] and helped organize a special session at the 1987 Mathematical Theory of Networks and Systems (MTNS) Conference in Phoenix, Arizona, June 15-19, 1987.

2. The principal investigator presented the invited paper [3] in a special session on singular systems at the 26*th* IEEE Conference on Decision and Control (CDC), Los Angeles, Dec. 9-11, 1987.

3. The principal investigator presented the invited paper [8] at the 1987 International Symposium on Singular Systems, Atlanta, GA, Dec. 4-6, 1987.

4. The principal investigator presented the invited paper [4] at the 88 IMACS World Congress on Scientific Computation, Paris, July 18-22, 1988.

5. The principal investigator presented the paper [11] and chaired a session at the 27*th* IEEE Conference on Decision and Control, Austin, Texas, Dec. 7-9, 1988.

6. The principal investigator presented the invited paper [13] at the 1989 American Control Conference, Pittsburgh, June 21-23, 1989.

7. The principal investigator presented the invited paper [14] at the IFAC Workshop on System Structure and Control, Prague, Czechoslovakia, Sept. 25-27, 1989.

8. The principal investigator gave an invited talk on the funded research at the Helsinki University of Technology, Helsinki, Finnland, Sept. 29, 1989.

9. The principal investigator has given several reports on project results in the numerical analysis seminar of the Center for Research on Scientific Computation at North Carolina State University.

10. The principal investigator gave an invited talk on the funded research at Clarkson University Electrical Engineering Department on May 3, 1989.

11. The principal investigator gave an invited talk on the funded research at Georgia Institute of Technology Electrical Engineering Department on March 9, 1989.

12. The principal investigator presented a paper on the funded research at the 1990 Conference on the Numerical Solution of Ordinary Differential Equations, Helsinki, Finnland, June 18-22, 1990.

13. The principal investigator gave two invited talks on the funded research at the Institut National de Recherche en Informatique et en Automatique (INRIA), Rocquenquort France, June 26 & 28, 1990.

14. The principal investigator was invited to organize a minisymposium at the SIAM Conference on Linear Algebra in Signals Systems & Control, San Francisco, California, June 7-9, 1990. The principal investigator organized a session and presented the paper [19] at the meeting.

15. The principal investigator organized a session on *New directions for implicit systems* at the 29*th* IEEE Conference on Decision and Control, Honolulu, Hawaii, Dec. 5-7, 1990. He also presented the paper [21] at this meeting.

16. W. J. Terrell gave a talk based on his thesis [20] at the Southeastern Differential Equations Conference, Blacksburg, Virginia, Nov. 17, 1990.

7.2 Other Meetings

As part of this research project, the principal investigator has attended and actively participated in the following meetings in addition to those described in Section 7.1.

1. The principal investigator attended the 1987 IEEE International Conference on Robotics and Automation, Raleigh, NC, March 31-April 3, 1987.

2. The principal investigator organized a minisymposium and chaired a session at the SIAM Conference on Control in the 90's, San Francisco, California, May 17-19, 1989. He was also on the Program Committee.

3. The principal investigator was an invited (and supported) participant at the NATO Workshop on The Real Time Simulation of Mechanical Systems, Snowbird, Utah, Aug. 7-11, 1989.

4. The principal investigator attended the 28th IEEE Conference on Decision and Control, Tampa, Florida, Dec. 13-15, 1989.

5. The principal investigator attended the Southeastern Differential Eaquations Conference, Blacksburg, Virginia, Nov. 16-17, 1990.

7.3 Consultative and Collaborative Activities

1. There has been substantial progress on some aspects off the numerical solution of singular systems. However, much of this material is highly technical in nature and not readily available to the numerical practitioner. Accordingly, the principal investigator and two collaborators have written the first monograph [12] on the numerical solution of singular systems which is aimed at those interested in scientific computation. (The only other book is highly theoretical, does not

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address many of the concerns we do, and is published in East Germany). This effort not only brings together much of the recent developments on implicit differential equations into a usable form for the first time, but contains many new results of the authors. The collaborators are

(1) Dr. Linda R. Petzold, computation group head, Lawrence Livermore Laboratories, Livermore, California.

(2) Dr. K. E. Brenan, applied mathematician, Aerospace Corporation, Los Angeles, California.

2. Dr. Kenneth D. Clark is a 1986 Ph.D. student of the principal investigator who was graduate study was supported in part by AFOSR-84-0240. The paper [1] is based on results originally done under AFOSR funding. Dr. Clark is currently a program manager in computational mathematics at the United States Army Research Office. He has continued to collaborate with the principal investigator particularly on questions involving problem structure and how it effects convergence of a numerical method.

3. Dr. Benjamin Leimkuhler is an Assistant Professor in the Mathematics Department at the University of Kansas. Dr. Leimkuhler and the principal investigator began the consideration of the effect of differentiation of constraints in [16]. This collaboration laid the groundwork for the more recent results in [17], [18].

4. Dr. Marek Rakowski is a visiting Assistant Professor at North Carolina State University. Dr. Rakowski and the principal investigator are examining if the results of Dr. Rakowski on realizations of time varying matrix functions can be used to explicitly compute the completion of a linear time varying control system (4).

5. Professor Nancy K. Nichols of the University of Reading, England, collaborated with the principal investigator and Dr. Terrell on [19].

8 Other References

[O1] K. E. Brenan and L. R. Petzold, *The numerical solution of higher index differential/algebraic equations by implicit Runge-Kutta methods*, Lawrence Livermore National Laboratory Report UCRL-95905, 1986.

[O2] L. Dai, *Singular Control Systems*, Lecture Notes in Control and Information Sciences, Springer-Verlag, New York, 1989.

[O3] B. J. Leimkuhler, L. R. Petzold, and C. W. Gear, *On obtaining a consistent set of initial values for a system of differential-algebraic equations*, University of Illinois Computer Science Report No. UIUCDCS-R-87-1344, 1987.

[O4] A. J. van der Schaft, *On realizations of nonlinear systems described by higher order equations*, Mathematical Systems Theory, 19 (1987), 239-275.